Section 6.1 Angles and Their Measure
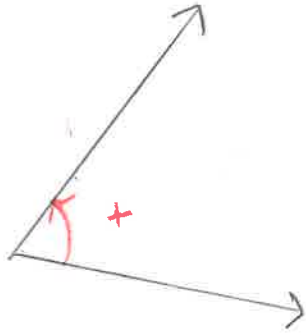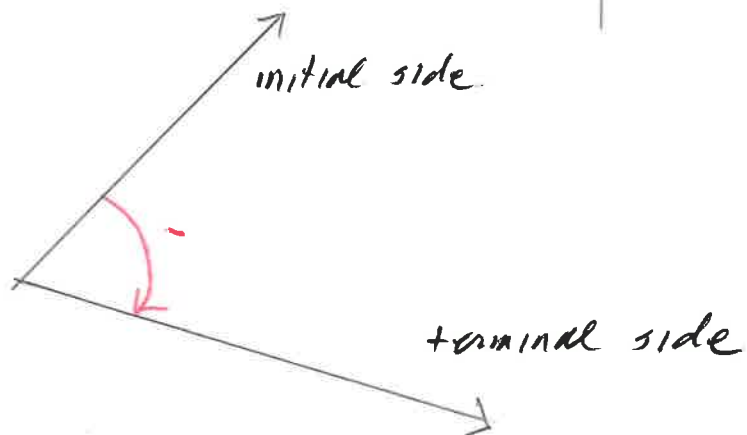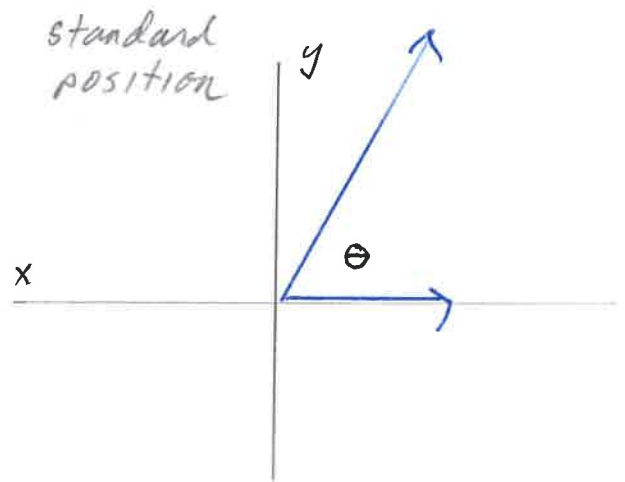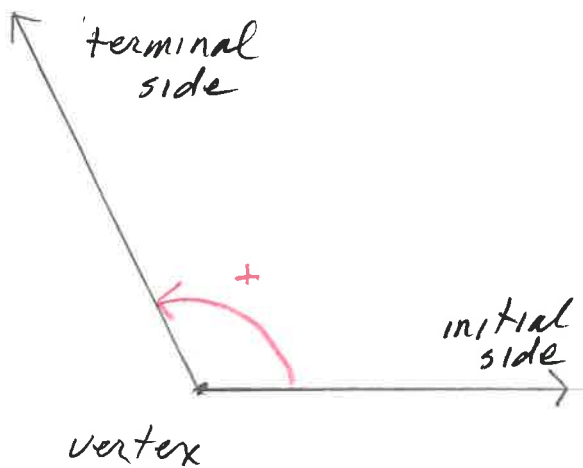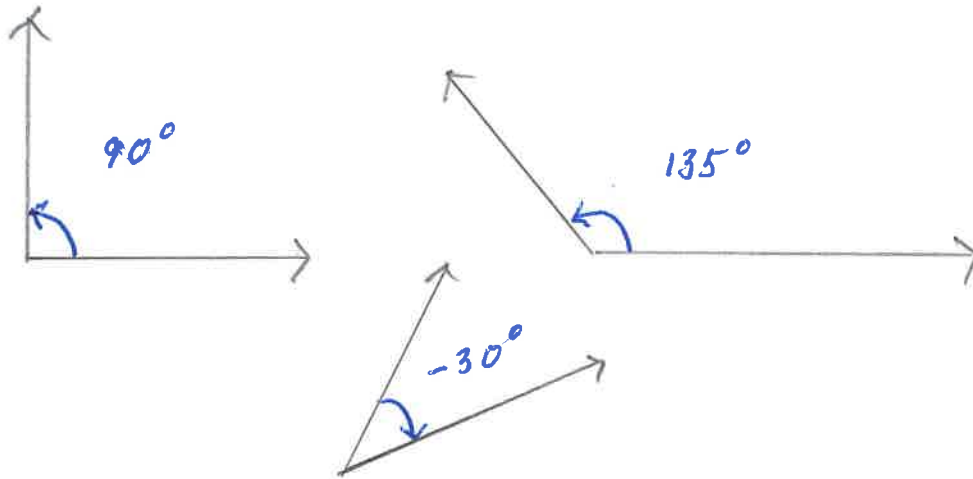
What is an angle?
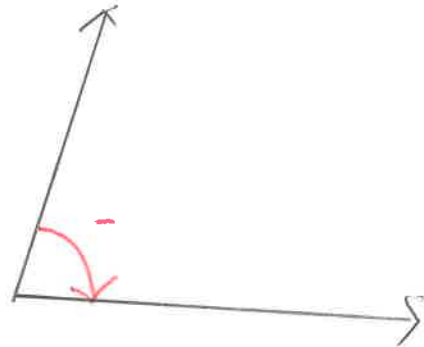
Make sure you are familiar with the following terms related to an angle: initial side, terminal side, vertex, standard position, positive measure, negative measure.

Measure of an Angle:

Degree Measure:

$90°$

$135°$

$-30°$
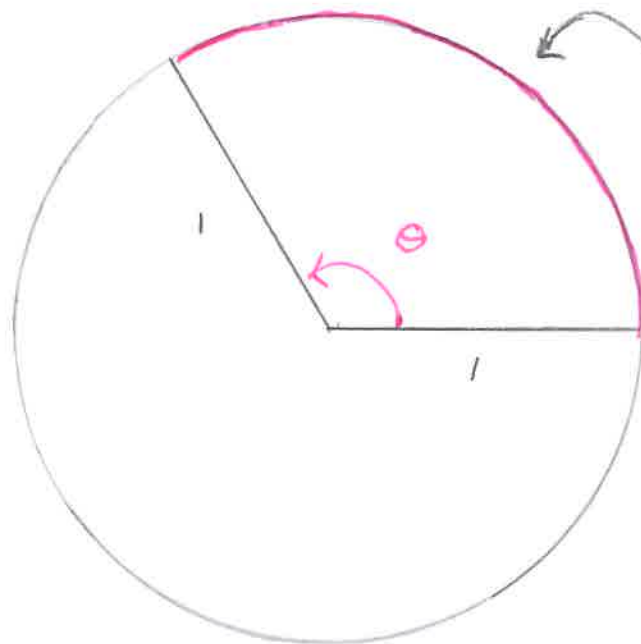
Radian Measure:

measure of θ in radians is the length of

θ

The circumference is 2π.

2π radians $= 360°$

π radians $= 180°$, 1 radian $= \left(\dfrac{180}{\pi}\right)°$

1 degree $= \dfrac{\pi}{180}$ radians.

Draw a picture of an angle with the following measure

3 radians
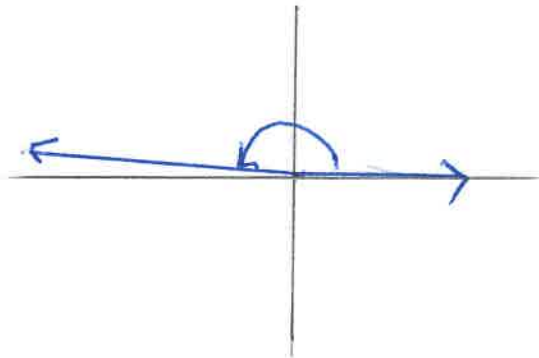
4.3 radians

$\frac{\pi}{3}$ radians

$-\frac{3\pi}{4}$ radians

7π radians

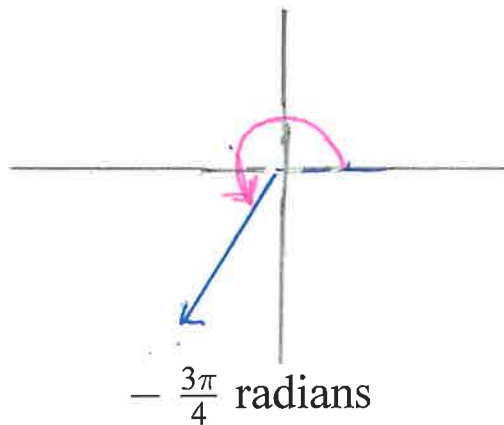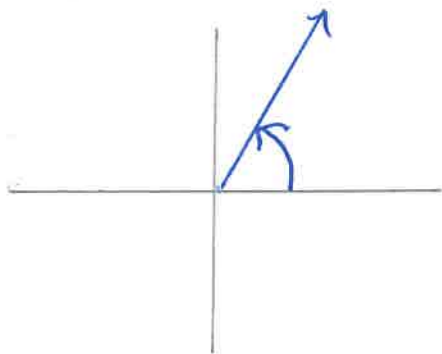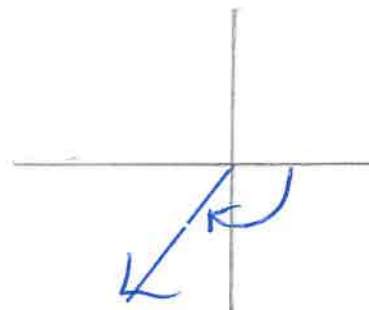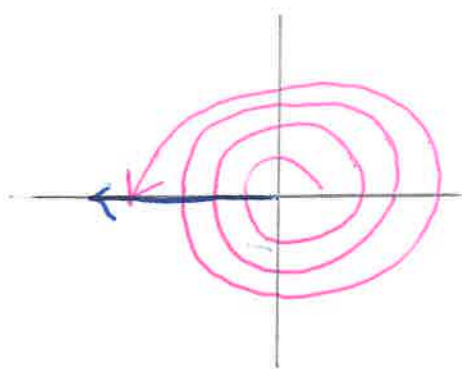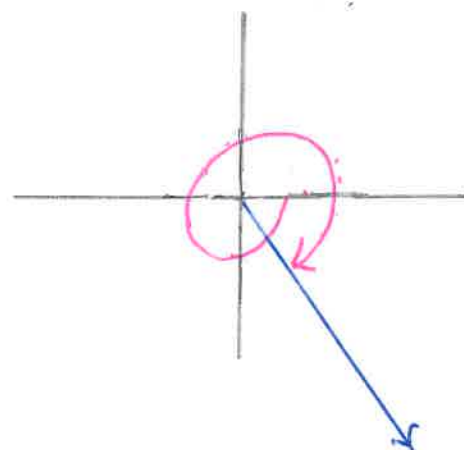### $-\frac{7\pi}{3}$ radians

Converting angles from degree measure to radian measure and vice-versa.

$$\pi \text{ radians} = 180° \quad , \quad 1 \text{ radian} = \left(\frac{180}{\pi}\right)°$$

$$1° = \frac{\pi}{180} \text{ radians}$$

An angle with a measure of $x°$ will have a measure of $x\frac{\pi}{180}$ radians.

An angle with a measure of x radians will have a measure of $(x\frac{180}{\pi})°$.

Example:
Convert 225° into radian measure.

$$225° , \quad 225 \text{ degrees} \cdot \frac{\pi \text{ radians}}{180 \text{ degrees}} = 3.93 \text{ radians}$$
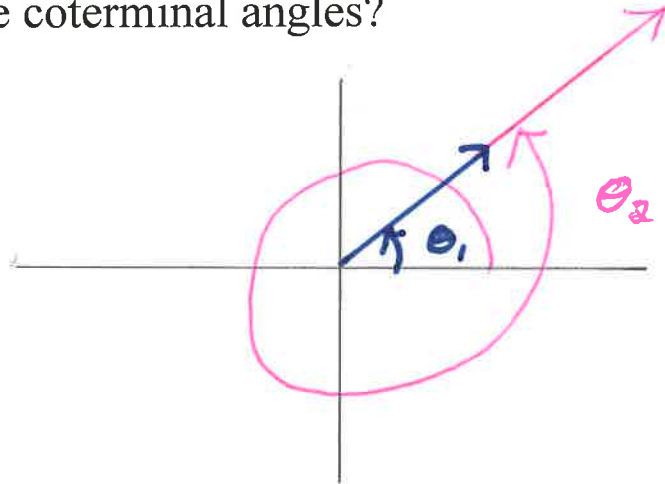$$(\text{two decimals})$$

Example:
Convert 3.3 radians into degree measure.

$$3.3 \text{ radians} \cdot \frac{180°}{\pi \text{ radians}} = 189.08 \text{ degrees}$$
$$= 189.08°$$

Example:
Convert $\frac{2\pi}{7}$ radians into degree measure.

$$\frac{2\pi}{7} \text{ radians} \cdot \frac{180 \text{ degrees}}{\pi \text{ radians}} = 51.43°$$

What are coterminal angles?

Find 3 angles that are coterminal to the angle with a measure of 120°.

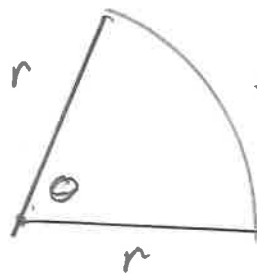$$480°, \quad -240°, \quad 840°$$

Find 3 angles that are coterminal to the angle with a measure of $\frac{3\pi}{4}$ radians.

$$\frac{3\pi}{4} + 2\pi = \frac{11\pi}{4}, \quad \frac{3\pi}{4} - 2\pi = \frac{-5\pi}{4}$$

$$\frac{3\pi}{4} + 4\pi = \frac{19\pi}{4}$$

How is the length of an arc of a circle related to the angle that subtends the arc?

$S = r\theta$

θ is measured in radians

If a circle has a radius of 12 cm, then find the length of the arc that is subtended by an angle with measure 2.2 radians.

$$S = r\theta \quad , \quad S = (12 \text{ cm})(2.2)$$

$$= 26.4 \text{ cm}$$

If a circle has a radius of 12 cm, then find the length of the arc that is subtended by an angle with measure 210°.

$$S = r\theta \quad , \quad S = (12 \text{ cm})\left(\frac{7\pi}{6}\right) = 43.98 \text{ cm}$$

θ must be measured in radians.

Commonly Encountered Angles:

30° or $\frac{\pi}{6}$ radians

60° or $\frac{\pi}{3}$ radians

45° or $\frac{\pi}{4}$ radians

90° or $\frac{\pi}{2}$ radians